AD-A009 231

LABORATORY INVESTIGATION OF THE SUNDSTRAND Q-FLEX ACCELEROMETER

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Warminster, Pennsylvania

4 April 1975

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NADC-75030-60	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER AD-A009231
4. TITLE (and Subtitle) Laboratory Investigation of the Sundstrand Q-Flex Accelerometer		5. TYPE OF REPORT & PERIOD COVERED Final Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) William F. Sapp		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Navigation Laboratory (Code 60) Naval Air Development Center Warminster, Pennsylvania 18974		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AIRTASK F21-234-601 Work Unit DV401
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Air Systems Command Department of the Navy Washington, D. C. 20361		12. REPORT DATE 4 April 1975
		13. NUMBER OF PAGES ~~25~~ 30
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Accelerometers
Navigation
Inertial Systems
Inertial Sensors

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The Sundstrand Q-Flex Accelerometer has undergone a short-term investigation to provide information for assessing the applicability of this accelerometer for strapdown inertial navigation systems.

The investigation included tests to determine model equation coefficients, the stability of the coefficients as a function of time and the sensitivity of the instrument to changes in environment. Particular emphasis was placed on the determination of thermal characteristics.

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SUMMARY

INTRODUCTION

A short-term performance evaluation was conducted on the Sundstrand Q-Flex Accelerometer, S/N 101, at NAVAIRDEVCEN (Naval Air Development Center) from 15 November 1974 to 14 January 1975. The accelerometer is the basic sensor used on the ATIGS (Advanced Tactical Inertial Guidance System). It was on loan from the ATIGS program for a period of three months and, during this time, a total of 202.5 hours of operation was accumulated.

The objectives of the abbreviated investigation program were to determine those basic performance characteristics which affect system performance and to identify particular error sources requiring further investigation. Particular emphasis was placed on a determination of the thermal characteristics.

The evaluation program consisted of tests to determine the accelerometer's model equation coefficients, the stability of the coefficients as a function of time and temperature storage, and the thermal characteristics of the instrument. Since thermal tests were of particular interest during this evaluation, three sensors were attached to the accelerometer for monitoring its temperature. The temperature sensors were attached to the top, the circumference, and the flange of the accelerometer. Figure 1 shows the placement of the sensors and the mount heater to which the accelerometer was attached. The normal operating temperature during the evaluation was 170°F (76.7°C).

SUMMARY OF RESULTS

A summary of the results obtained on the Q-Flex accelerometer is given in Table I.

TABLE I

SUMMARY OF RESULTS

Accelerometer Parameter	
Scale Factor (K_1)	0.2696880 volts/g
Bias (K_o/K_1)	-182.5μg
Second Order Nonlinearity (K_2/K_1)	Insignificant
Third Order Nonlinearity (K_3/K_1)	24.0$\mu g/g^3$
Cross Coupling (K_{oi}/K_1)	23.5$\mu g/g^2$
Stability	
Stability at Zero g	4.3μg (1σ)
Stability at One g	20.4μg (1σ)

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TABLE I (CONTINUED)

	Scale Factor Mean Value (volt/g)	Scale Factor Std Dev (ppm)	Bias Mean Value (μg)	Bias Std Dev (μg)
Short-Term Stability (No cool down)	0.269745	5.0	-232.8	3.0
Cool down Repeatability				
Storage at 23°C	0.269817	140.8	-148.9	19.7
Storage at 0°C	0.269916	108.0	-104.6	25.2

Thermal

Operating Temperature Sensitivity

Temperature range of 74°C to 80°C

$K_1 = 131.7$ ppm/°C

$K_0/K_1 = 23.4$ μg/°C

Temperature range of 27°C to 77°C

$K_1 = 118.8$ ppm/°C

$K_0/K_1 = 26.8$ μg/°C

Warm-Up

Warm-up achieved in 12.5 minutes from 23°C and 0°C for tests conducted at NAVAIRDEVCEN

Warm-up time is dependent on heating rate applied to the instrument

CONCLUSIONS

The performance parameters that were determined during this investigation were all within the ATIGS Accelerometer Procurement Specification (Reference a) except for the long-term (60 days) scale factor stability. The sensitivity of the instrument to changes in the operating temperature was apparent throughout the evaluation and while this sensitivity was within specification, it is suggested that a design modification such as a thermal compensation network could reduce the temperature sensitivity near the operating temperature and further improve performance. The repeatability of the accelerometer output observed in the warm-up tests indicates that thermal modelling could be an effective method of error compensation on the system level.

RECOMMENDATIONS

None. This report is for information only.

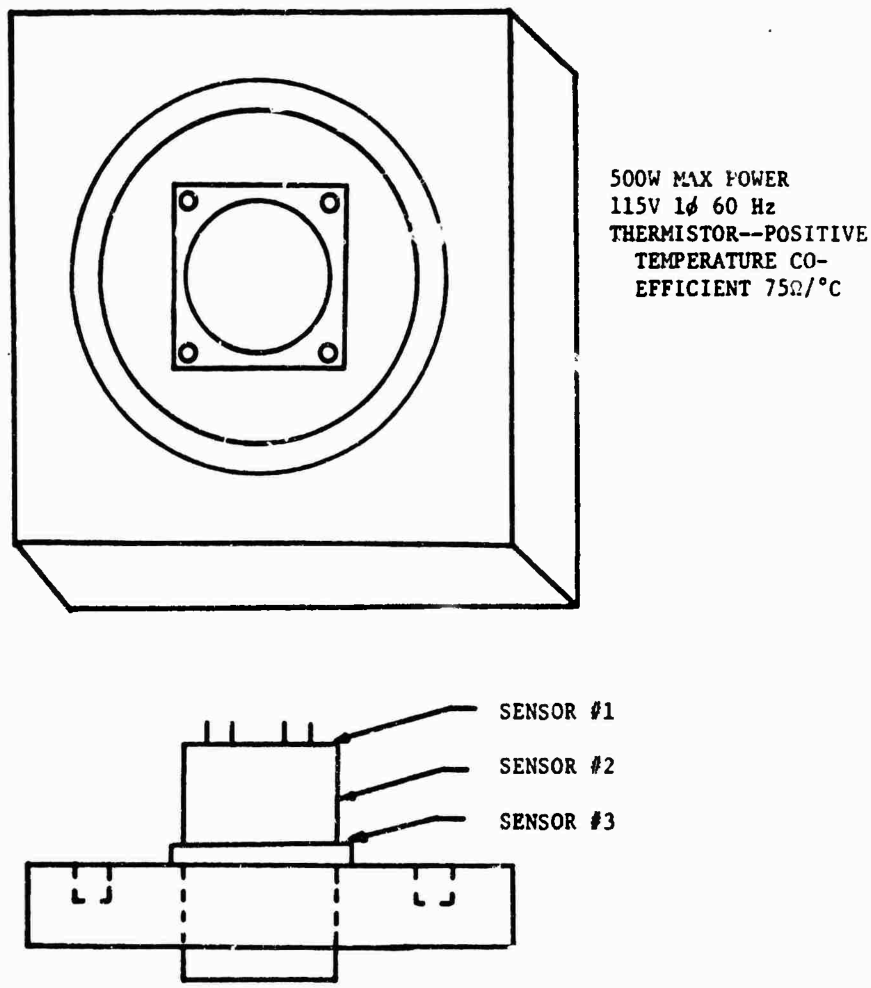

Figure 1. Mounting Block and Temperature Sensor Placement

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LIST OF SYMBOLS

A	Accelerometer Output
K_o	Null Offset (Bias)
K_1	Scale Factor
K_n	n^{th} Order Non-linearity
Δ_o, Δ_p	Misalignment Angle of Input Axis
K_{oi}, K_{pi}	Cross Coupling Due to Acceleration Along o, p Axes
a_i, a_o, a_p	Acceleration Along the Input, Output, and Pendulum Axes, respectively
σ	Standard Deviation
μg	1×10^{-6} g
ppm	Parts Per Million

DESCRIPTION

The Sundstrand Q-Flex Accelerometer is a single axis, pendulous type, force rebalance accelerometer with the sensor and force rebalance electronics in one unit. The size of the unit is 1.0 inches in diameter and 1.5 inches high, and its weight is 2.6 ounces.

The pendulous proof mass, two parallel flexure supports and an outer clamping ring are made from a single disc of amorphous quartz which is shaped by air abrasion and chemical milling to form an integral structure.

When the accelerometer is subjected to an acceleration along its sensitive axis, a capacitance bridge detects the resulting displacement of the proof mass and provides an electrical error signal to the servo for capturing the pendulum. Capture is accomplished by providing current to the torquer coils attached to the pendulum which interact with a magnetic field to return the proof mass to its center position. The DC current to the torquer coils required to maintain the proof mass at its center position is directly proportional to the acceleration being sensed.

TEST INSTRUMENTATION

The Q-Flex accelerometer was positioned with respect to the gravity field by an Ultradex Indexing Head which can be set in one degree increments with an accuracy of 0.25 seconds of arc. The output voltage of the accelerometer was measured using a Type 9144 Guildline Potentiometer for all tests except warm-up tests. During warm-up tests, the accelerometer output was measured using a Hewlett Packard 2401 C digital voltmeter because of the need to measure the changing accelerometer output every 10 seconds. A Hewlett Packard 562 A printer was used in conjunction with the digital voltmeter to record the accelerometer measurements. Two Digitec Thermometers were used to measure accelerometer temperature and a Digitec Printer was used to record the accelerometer temperature in synchronization with the measurement of the accelerometer output. A modified Tenny Temperature Chamber with an opening port in the door was used in certain tests to completely enclose the accelerometer while on the dividing head. This permitted storage of the accelerometer at 0°C for cool down bias repeatability tests and warm-up tests without disturbing its initial alignment.

TEST PROGRAM

MODEL EQUATION

The test program for the Sundstrand Q-Flex Accelerometer consisted of determining certain characteristics of the accelerometer and measuring the stability and sensitivity of the model equation coefficients as a function of time and environment. The model equation employed was a mathematical series which relates the output of the accelerometer to the components of acceleration applied parallel and normal to the accelerometer input axis. The model equation expresses the accelerometer output as:

$$A = K_0 + K_1a_i + K_2a_i^2 + K_3a_i^3 + K_4a_i^4 + K_5a_i^5 + K_6a_i^6 + K_7a_i^7 + \Delta_pa_p + \Delta_oa_o + K_{pi}a_pa_i + K_{oi}a_oa_i$$

in which

A	=	Accelerometer Output (Output Units)
K_o	=	Bias (Output Units)
K_1	=	Scale Factor (Output Units/g)
K_n	=	n^{th} Order Non-linearity (n= 2,3,4,5,6,7) (Output Units/g^n)
Δ_o, Δ_p	=	Misalignment Angle of Input Axis (Radians)
K_{oi}, K_{pi}	=	Cross Coupling Due to Acceleration Along o, p Axes (Output Units/g^2)
a_i, a_o, a_p	=	Acceleration Along Input, Output, and Pendulum Axes, Respectively (Units of g)

The model equation coefficients were determined using both the 72-Point Test and the 6-Point Test.

THE 72-POINT TEST

The purpose of the 72-Point Tests is to accurately determine all the coefficients of the model equation. These tests are usually conducted at points in the test program where changes in the model equation coefficients are most likely to occur. Since the test program was shorter than that usually conducted, only two 72-Point Tests were performed; one at the beginning of the test program and the other at the end of the test program.

The output data from the Sundstrand accelerometer was Fourier analyzed to determine the zero, first, second, and other higher order harmonics. Since the Fourier analysis is an approximation of the accelerometer output, there is a small residual or difference between the Fourier curve fit and the recorded output. For a coefficient to be significant and be included in the calculation of the model equation coefficients, it must be significantly greater than the residual, σ_R, of the Fourier fit. The residual, σ_R, is the residual for the best fit model because it is calculated using the deviations of the data from a model that contains all the significant Fourier coefficients. In addition to determining the residual for the best fit model, a total non-linearity, σ_T, is also calculated. The total non-linearity is calculated using the deviations of the data from a linear model which uses only the A_o and B_1 Fourier coefficients.

The model equation coefficients are calculated from the significant Fourier coefficients, and they are then statistically tested to determine if they are significant. An explanation of the 72-Point Test, which includes the relationship between the model equation coefficients and the Fourier coefficients, is given in Appendix A.

THE 6-POINT TEST

The 6-Point Test was used in stability and sensitivity tests where the coefficients had to be determined more than once a day. It was impractical in these tests to use the 72-Point Test. In the 6-Point Test, the coefficients of the accelerometer were determined by measuring the output of the accelerometer at 6 selected levels of acceleration and using this data to calculate the model equation coefficients. An explanation of the 6-Point Test is given in Appendix B.

ACCELEROMETER CHARACTERISTICS

The model equation coefficients determined from the two 72-Point Tests performed on the Q-Flex accelerometer are summarized in Table II.

TABLE II

	Initial Test	Final Test
K_1	0.2696880 volts/g	0.2698826 volts/g
K_o/K_1	-182.5 μg	-6.5 μg
K_2/K_1	Insignificant	Insignificant
K_3/K_1	24.0 $\mu g/g^3$	38.2 $\mu g/g^3$
Δ_o/K_1	-10.3 μ radians	-14.9 μ radians
K_{oi}/K_1	23.5 $\mu g/g^2$	11.4 $\mu g/g^2$

Non-linearity terms above K_3 were zero

Since the two 72-Point Tests were performed at the beginning and the end of the program, they indicate the stability of the scale factor, bias and other model equation coefficients over the 3-month period of time. The changes in the scale factor and bias determined from the two 72-Point Tests were 721 ppm and 176.0 μg respectively.

Plots of the deviations from the best model and linear model for the initial test are shown in Figure 2. The residual, σ_R, for the best fit model was 5.0 μg and for the linear model the total non-linearity, σ_T, was 13.7 μg. In the final test, σ_R was 3.1 μg and σ_T was 15.0 μg.

STABILITY TESTS

This phase of the investigation included tests to determine the stability of the accelerometer output at zero g and one g, the short-term stability of the model equation coefficients, and the repeatability of the model equation coefficients with cool down to 23°c and 0°C.

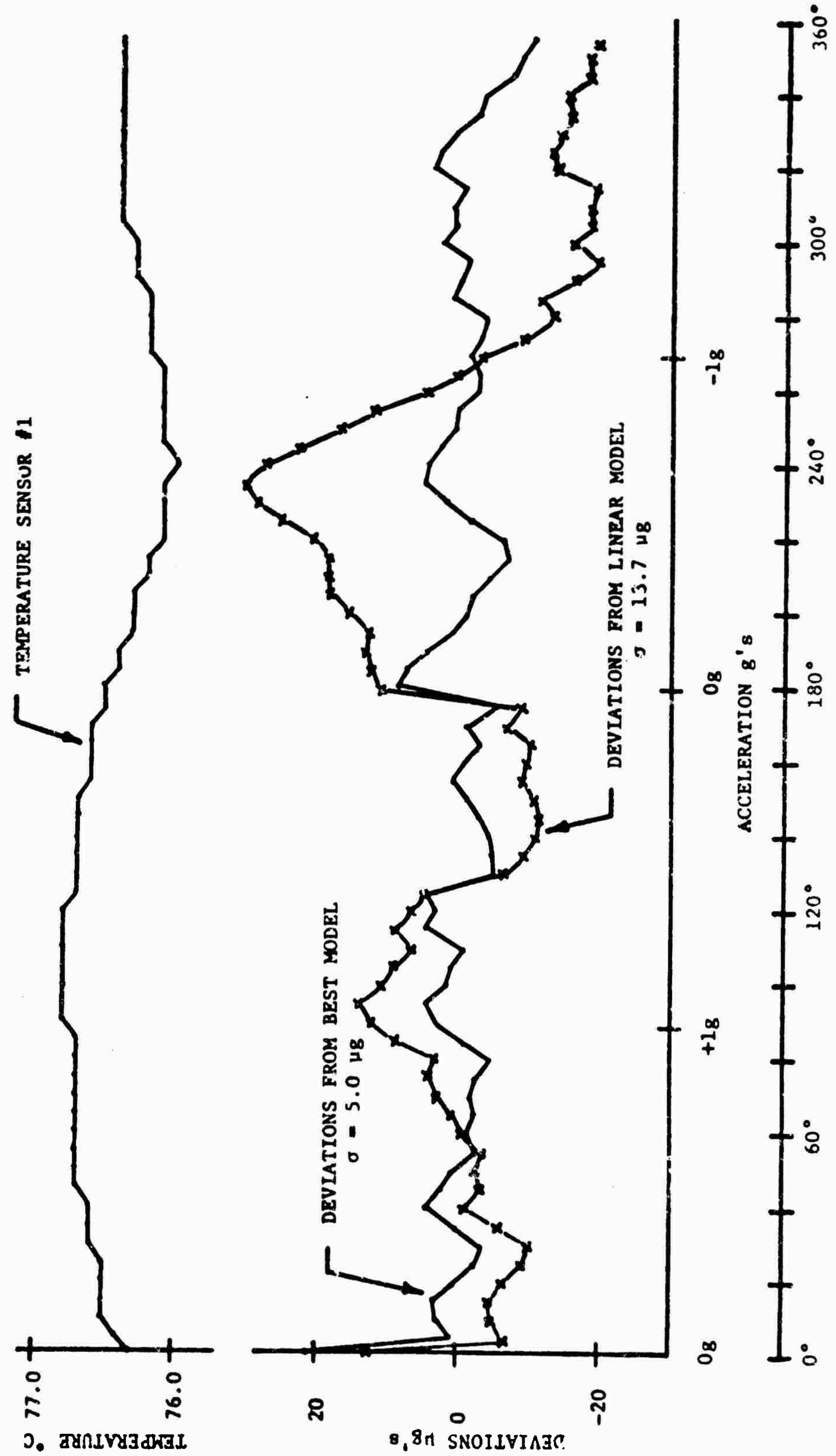

Figure 2. Deviations From Best Model and Linear Model

STABILITY AT ZERO g

The purpose of this test is to determine the short-term stability of the accelerometer output with zero input acceleration. Prior to the start of the test, the accelerometer was operated for 2 hours to allow for instrument warm-up and stabilization. The indexing head was then positioned for zero input acceleration to the instrument and the accelerometer output was recorded every 5 minutes for a period of 6 hours. The temperature of the instrument was also recorded each time a measurement of the accelerometer output was made and plots of accelerometer output and temperature for each measurement are shown in Figure 3. The total change in temperature over the 6-hour period was 0.5°C, and it is apparent from the plots of temperature and accelerometer output that changes in the accelerometer output are correlated with changes in temperature. Temperature sensitivity tests, which will be discussed later, show that the sensitivity of bias to temperature is 23.4 μg/°C. The standard deviation of the accelerometer output over the 6-hour period was 4.3 μg with the temperature controlled to 0.5°C.

STABILITY AT ONE g

The procedure used for conducting this test is identical to that used to determine the stability at zero g except that the instrument is positioned to sense an acceleration of one g. Plots of accelerometer output and accelerometer temperature for each measurement made during the one g test are shown in Figure 4.

The correlation between accelerometer output and accelerometer temperature is also apparent in this test, and it can be seen that the sensitivity to temperature is greater at one g than at zero g. The larger sensitivity to temperature exhibited at one g is in agreement with temperature sensitivity tests which show that the scale factor sensitivity is 131.7 ppm/°C compared to the bias sensitivity of 23.4 μg/°C. The standard deviation for the accelerometer output measurements during the 6-hour one g test was 20.4 μg with temperature controlled to 0.5°C.

SHORT-TERM STABILITY

The stability of the model equation coefficients over a 6-hour period was measured using a 6-Point Test. A warm-up and stabilization period of 2 hours was allowed before the initial 6-Point Test was performed. The 0°, 45°, 90°, 135°, 180°, and 270° test positions established during the initial test were utilized for each 6-Point Test conducted each hour for a period of 6 hours.

From these tests, values of K_0/K_1, K_1, K_2/K_1, K_3/K_1, Δ_0/K_1, and K_{oi}/K_1 were calculated to determine if any significant trends or changes could be observed in any of the coefficients. The temperature of the instrument was also recorded each time a 6-Point Test was conducted. Temperature changes were held to 0.1°C over the entire test period.

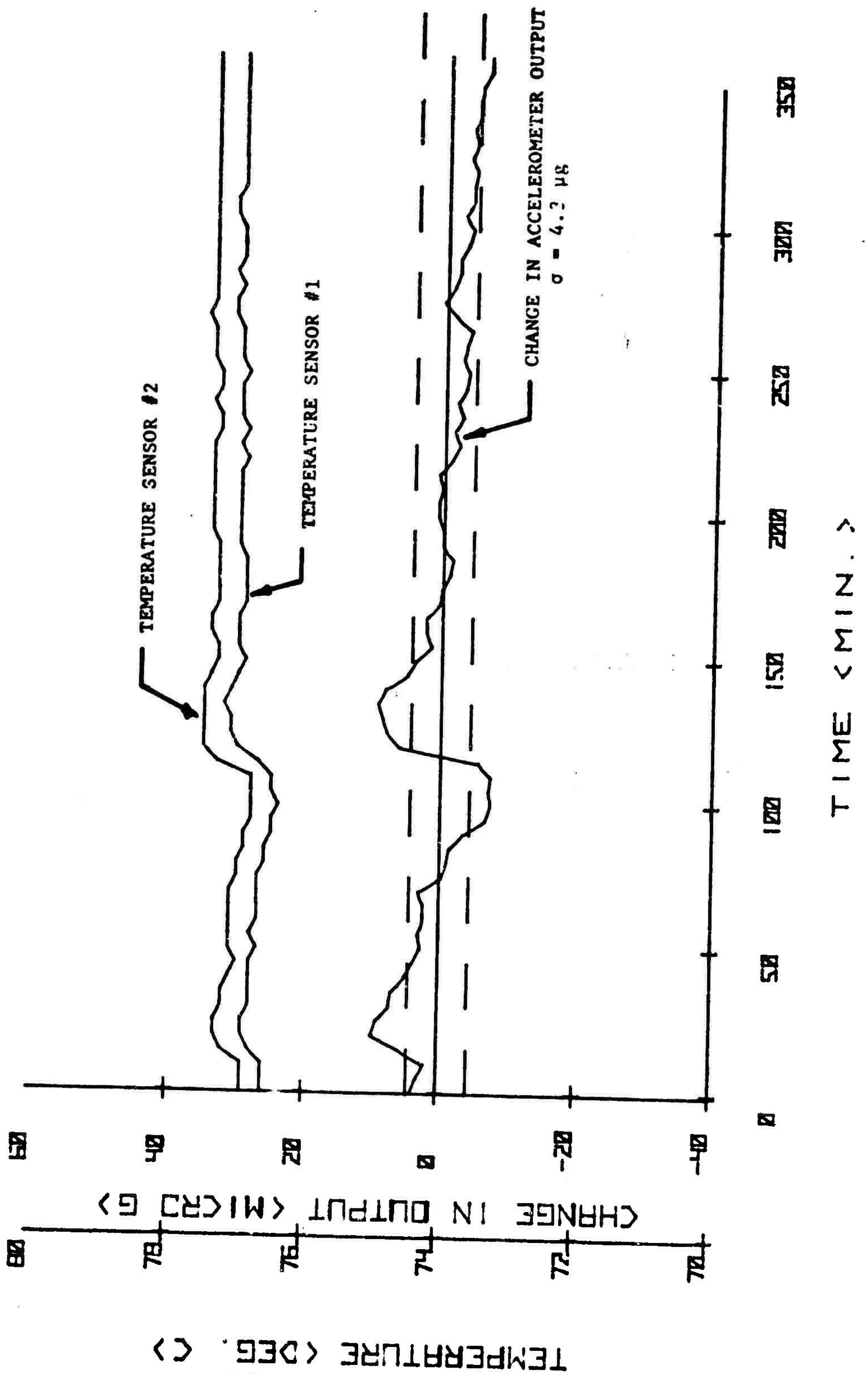

Figure 3. Stability at Zero g

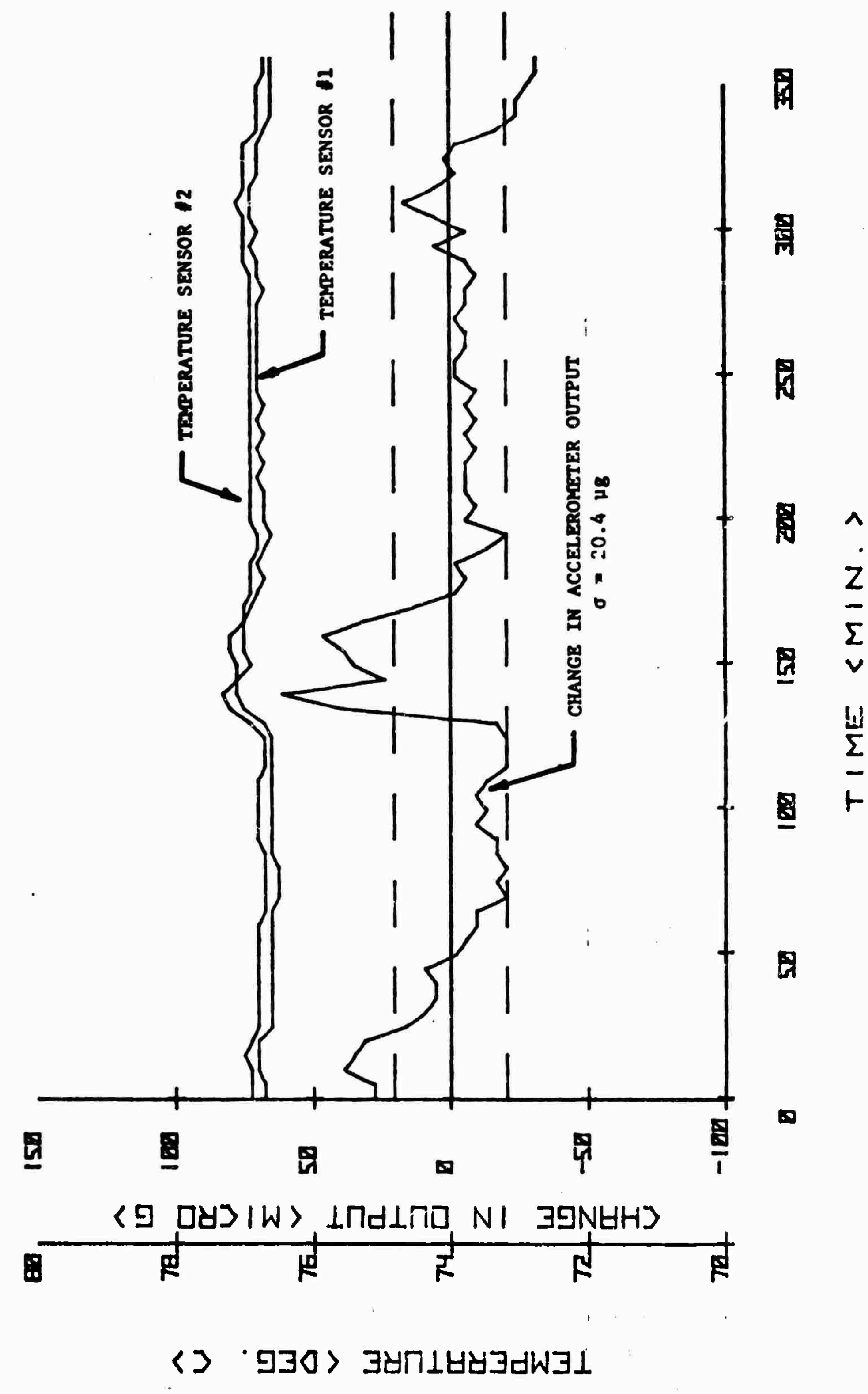

Figure 4. Stability at One g

The stability values of the scale factor (K_1) and bias (K_0/K_1) over the 6-hour period were 5.0 ppm (1σ) and 3 μg (1σ), respectively.

A plot of scale factor and bias along with accelerometer temperature are shown in Figure 5. These tests show that the scale factor and bias are stable over a continuous operating period if the temperature can be well controlled.

COOL DOWN REPEATABILITY

The purpose of the cool down repeatability test is to determine the repeatability of the model equation coefficients with shutdown and cool down of the instrument to 23°C and 0°C.

The model equation coefficients were determined each day during this series of tests by performing a 6-Point Test after warm-up of the instrument was complete. For the first series of tests, the accelerometer was shut down and stored at 23°C between each period of operation until a total of 10 measurements of the model equation coefficients were obtained. The second series of tests included 12 measurements of the model equation coefficients with shutdown and cool down of the instrument to 0°C between each period of operation. The model equation coefficients determined for both series of tests were plotted to see if any significant trends or changes could be observed in any of the coefficients. Figure 6 shows a plot of the scale factor (K_1) and bias (K_0/K_1) for each storage temperature.

The repeatability of the scale factor and bias with storage at 23°C was 140.8 ppm (1σ) and 19.7 μg (1σ), respectively. With storage at 0°C, the repeatability of the scale factor was 108.0 ppm; and the bias repeatability was 25.2 μg. During the middle of the series with storage at 0°C, the bias decreased from -137 μg to -67 μg over a 15-day period; but by the end of the test period, appeared to be changing randomly.

In addition to the repeatability of the scale factor and bias at each storage temperature, changes in the mean scale factor and mean bias from storage at 23°C to 0°C were determined. The change in the mean scale factor with storage at 23°C and 0°C was 329 ppm, and the change in the mean bias between 23°C and 0°C was 44.3 μg.

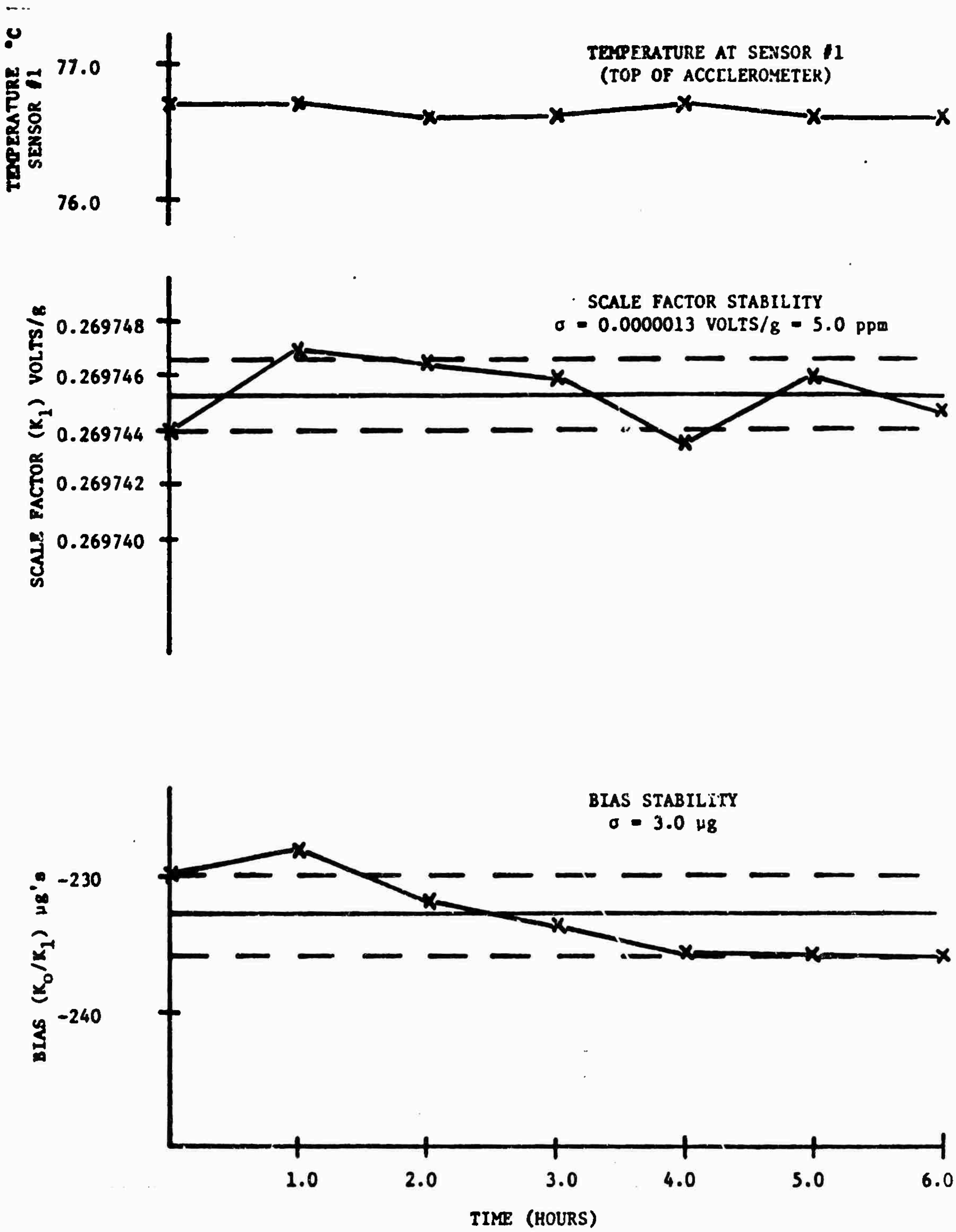

Figure 5. Short-Term Stability

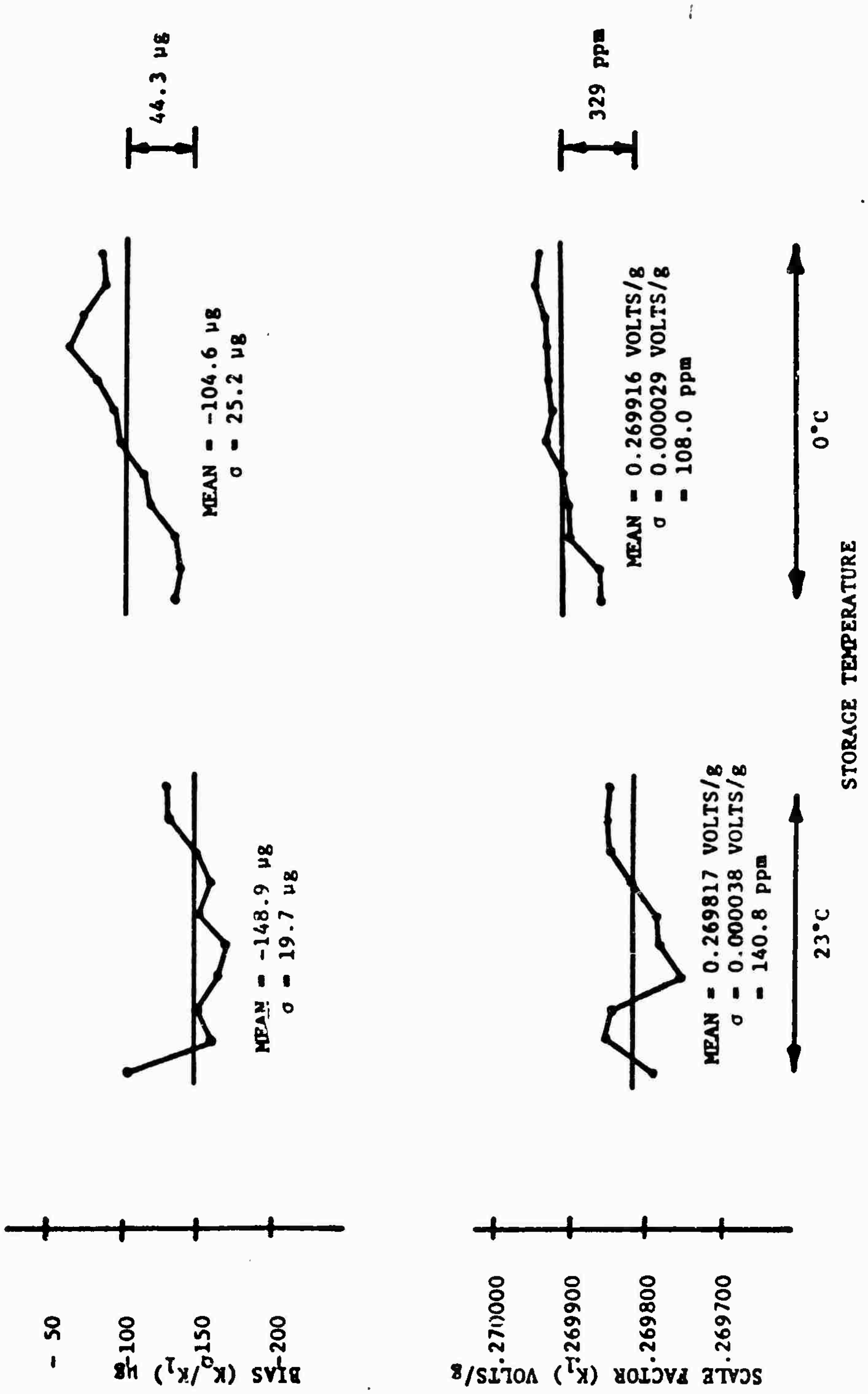

Figure 6. Cool Down Repeatability

TABLE III

STABILITY PERFORMANCE SUMMARY

Stability at Zero g	4.3 μg (1σ)
Stability at One g	20.4 μg (1σ)

	Scale Factor		Bias	
	Mean Value (volt/g)	Std Dev (ppm)	Mean Value (μg)	Std Dev (μg)
Short-Term Stability				
No cool down	0.269745	5.0	-232.8	3.0
Cool down Repeatability				
Storage at 23°C	0.269817	140.8	-148.9	19.7
Storage at 0°C	0.269916	108.0	-104.6	25.2

THERMAL TESTS

Thermal tests were conducted on the Q-Flex accelerometer to determine instrument sensitivity to changes in operating temperature, and instrument warm-up characteristics with prior storage and turn-on at 23°C and 0°C.

OPERATING TEMPERATURE SENSITIVITY

The purpose of this test is to determine the sensitivity of the model equation coefficients to operating temperature. The results obtained from this test will indicate how precise the operating temperature must be controlled in order that instrument performance is not affected by changes in temperature. Tests were conducted over a narrow temperature range from 74.0°C to 80.0°C (76.7°C nominal operating temperature) and a broad temperature range, 27.5°C to 76.7°C.

At each of the selected temperatures for both tests, the instrument was allowed to stabilize for approximately 15 minutes before a 6-Point Test was conducted. The model equation coefficients determined from each test were analyzed by the method of least squares to determine the sensitivity of each model equation coefficient to temperature.

Both of the tests that were conducted show a linear correlation between scale factor (K_1) and temperature, and bias (K_0/K_1) and temperature. The sensitivities of the scale factor and bias to operating temperature over the range of 74.0°C to 80.0°C were 131.7 ppm/°C (73.2ppm/°F) and 23.4 μg/°C (13.0 μg/°F), respectively. Over the temperature range of 27.5°C to 76.7°C, the scale factor sensitivity was 118.8 ppm/°C (65.7 ppm/°F) and the bias sensitivity was 27.0 μg/°C (15.0 μg/°F). The results of both tests are presented in Figures 7 and 8, and they demonstrate that the operating temperature must be controlled to 0.1°C to achieve the performance that was demonstrated in the short-term stability test.

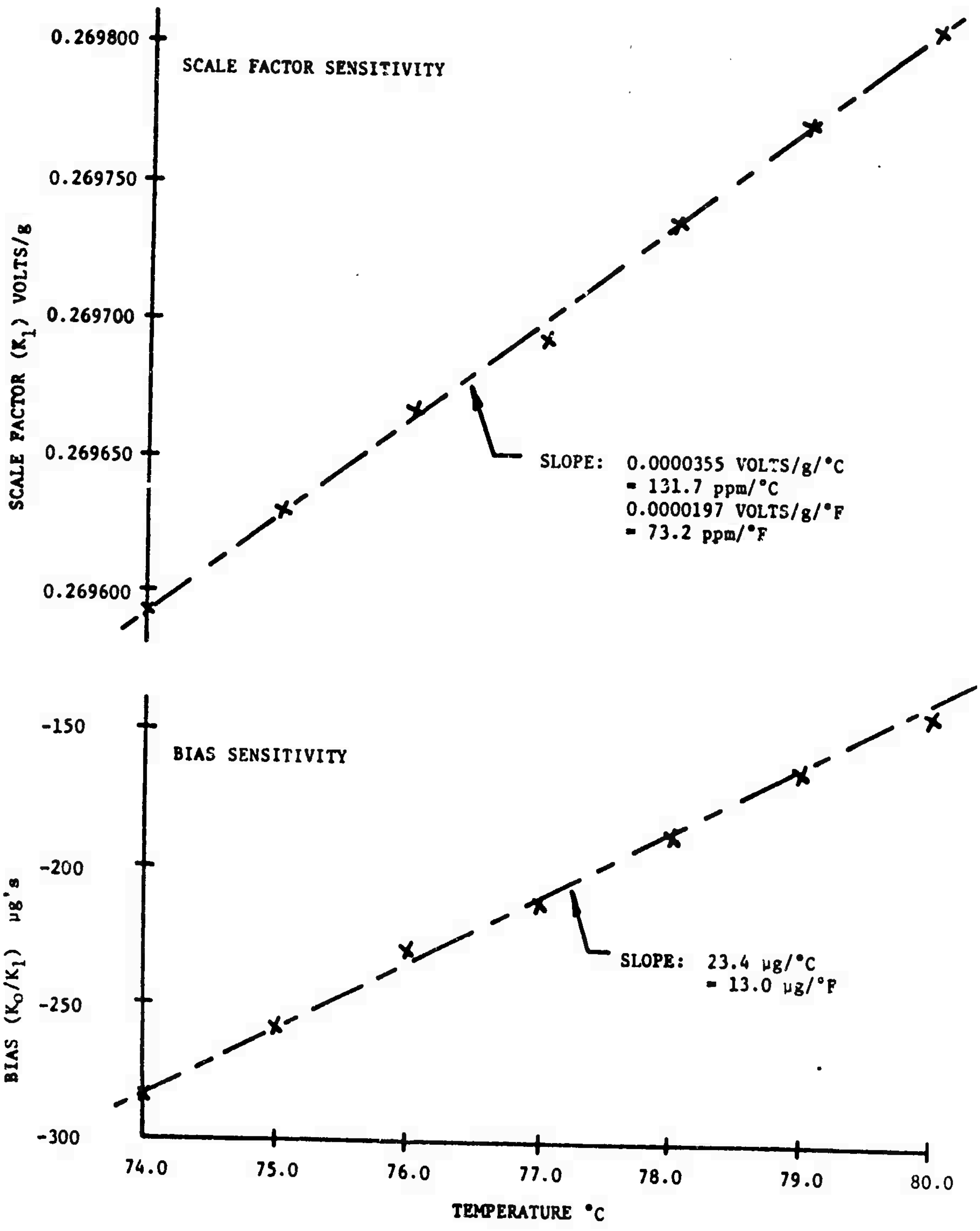

Figure 7. Operating Temperature Sensitivity (74.0°C to 80.0°C)

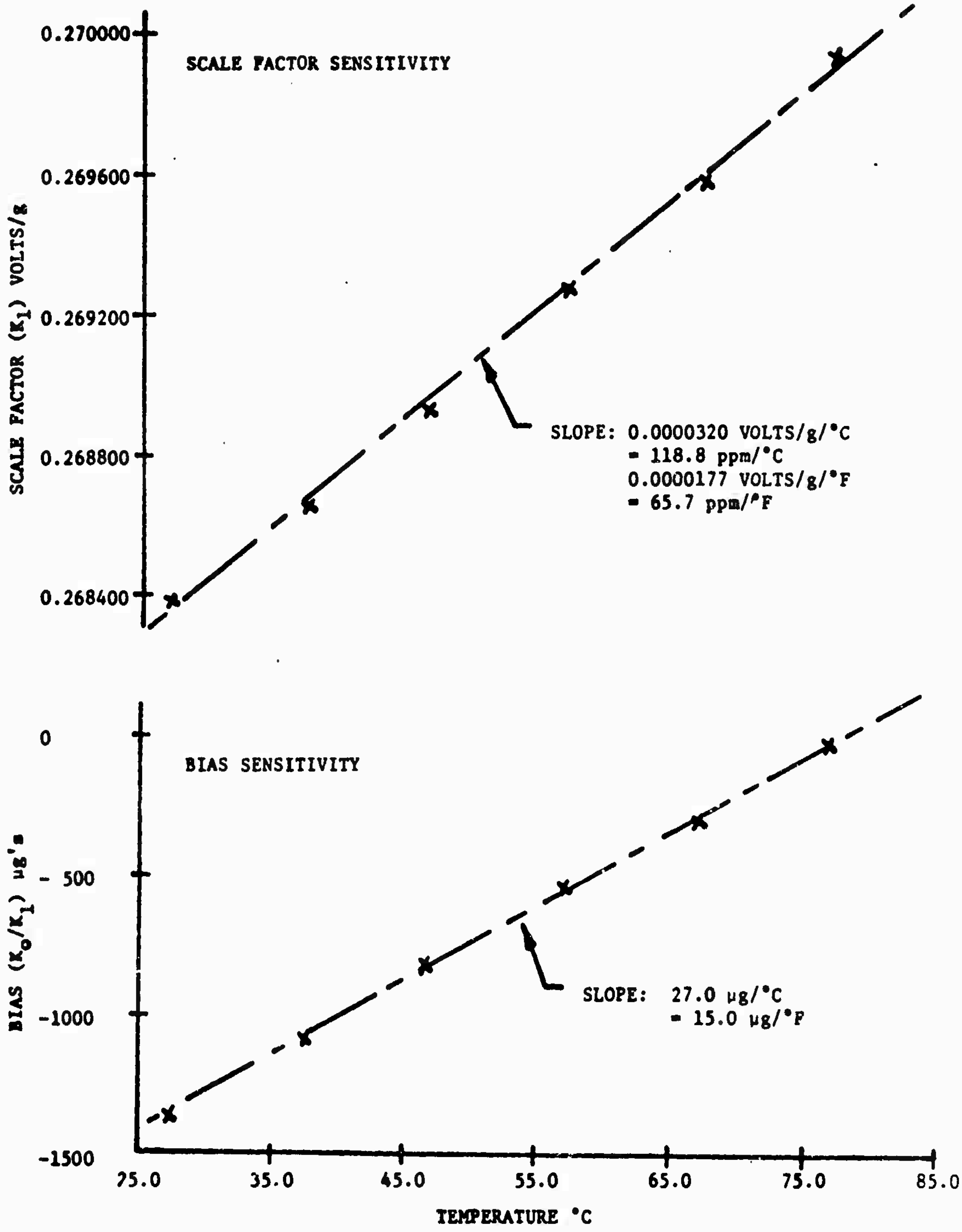

Figure 8. Operating Temperature Sensitivity (27.5°C to 76.7°C)

WARM-UP TESTS

Warm-up tests were conducted on the Q-Flex accelerometer to determine the time required for the instrument to reach a stabilized operating condition with prior storage and turn-on at 23°C and 0°C, and also to determine the repeatability of the accelerometer warm-up profile from these temperatures.

Each warm-up test consisted of a temperature soak of at least 4 hours at the storage temperature, followed by the simultaneous application of power to the mount heater and accelerometer. Once power was applied to the heater and accelerometer, the temperatures of the top (sensor #1) and flange (sensor #3) of the accelerometer, and the accelerometer output were recorded every 10 seconds for a period of one hour.

Eight warm-up tests were conducted from each storage temperature and every effort was made to repeat the heating profile during each test so that the repeatability of the accelerometer output during warm-up could be determined.

The accelerometer warm-up profiles obtained from each test conducted from 23°C are presented in Figure 9 along with the heating profile used for those tests. Figure 10 shows the accelereometer warm-up profiles and the heating profile for the series of warm-up tests from 0°C. The heating profiles are those obtained from temperature sensors #1 and #3. The heating rate of the accelerometer as measured by sensor #1 was 11 deg C/min for the first 3 minutes and then reducing to 1 deg C/min for the next 5 minutes for the warm-up tests from 23°C. From 0°C the heating rate measured by sensor #1 was 18 deg C/min for the first 3 minutes and then reducing to 1 deg C/min for the next 5 minutes.

The shape of the accelerometer warm-up profiles for the 8 tests conducted from 23°C are very repeatable and are directly correlated to the heating profile generated by sensor #1. The accelerometer warm-up profiles are not exactly duplicated because the level at which the accelerometer output stabilizes is a reflection of any change in accelerometer bias. If the accelerometer output was thermally modeled for error compensation in a system application, the major uncertainty in predicting accelerometer output as a function of temperature during warm-up would be the bias repeatability.

The tests conducted from 23°C and 0°C both show that a stable accelerometer output is achieved almost simultaneously with warm-up of the instrument to its normal operating temperature. The time required to reach a stabilized operating condition was approximately 12.5 minutes for both series of tests because the time required to reach the normal operating temperature was the same for tests conducted from 23°C and 0°C. It appears that warm-up can be accomplished more rapdily if the normal operating temperature is achieved in a shorter length of time.

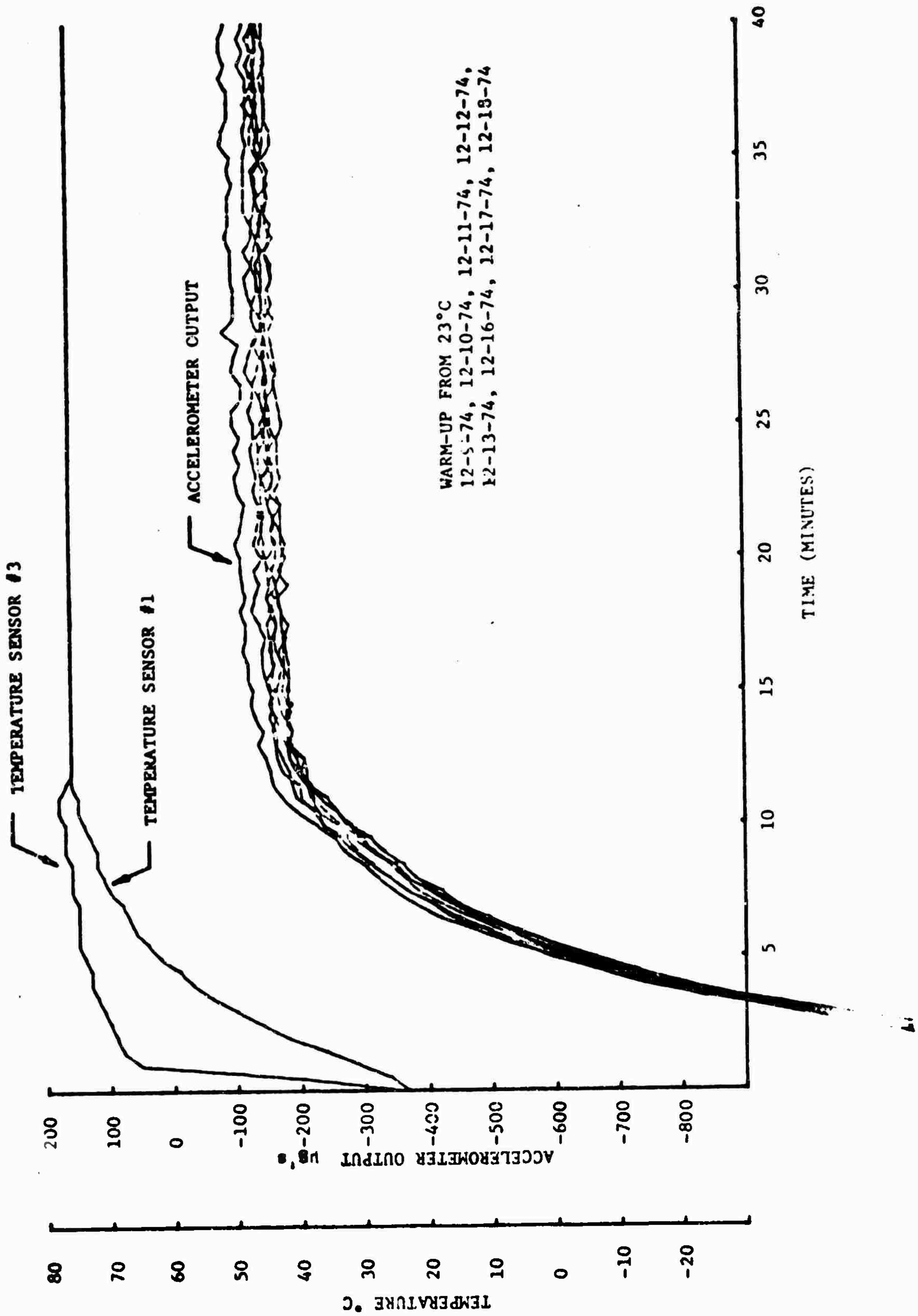

Figure 9. Warm-Up From 23°C

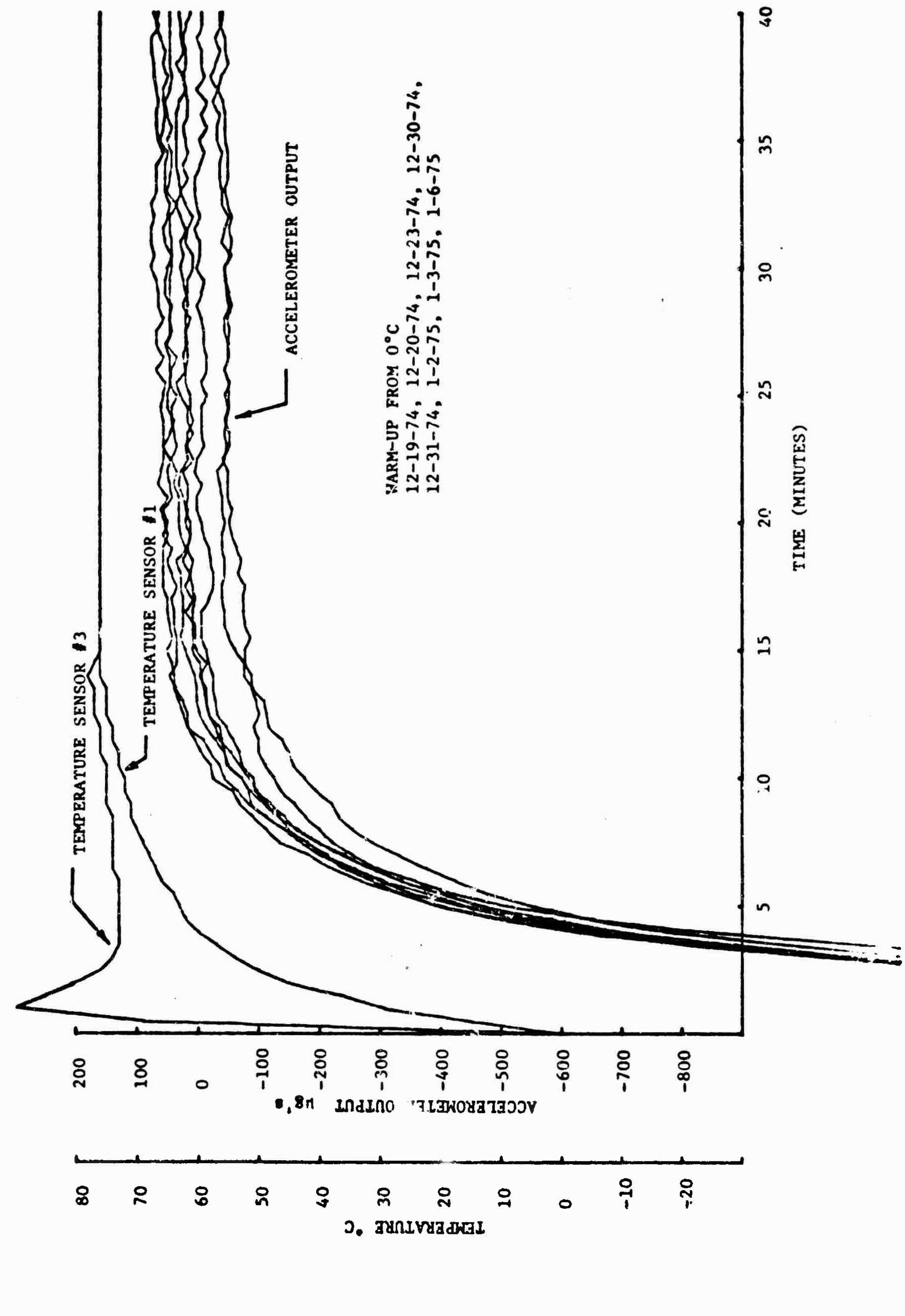

Figure 10. Warm-Up From 0°C

REFERENCES

(a) ATIGS Accelerometer Procurement Specification R-ED 21500 of 17 October 1974

THE 72 POINT TEST

The output of an accelerometer is theoretically proportional to the component of acceleration along the input axis. However, the realistic output is affected by such effects as null offset, non-linearity, cross coupling and misalignment. The 72 point test is employed to determine the accelerometer performance parameters with a low degree of statistical uncertainty.

The output of an accelerometer may be described by assuming a model equation.

$$(1)\quad A = K_o + K_1 a_i + K_2 a_i^2 + K_3 a_i^3 + K_4 a_i^4 + K_5 a_i^5 + \Delta_o a_o + \Delta_p a_p + K_{oi} a_o a_i + K_{pi} a_p a_i$$

in which A = accelerometer output (output units)

K_o = null offset (bias) (output units)

K_1 = scale factor (output units/g)

K_n = n^{th} order non-linearity (n = 2, 3, 4, or 5) (output units/g^n)

Δ_o, Δ_p = misalignment angle of input axis (radians)

K_{oi}, K_{pi} = cross coupling due to acceleration along o, p axes (output units/g^2)

a_i, a_o, a_p = acceleration along the input, output and pendulum axes, respectively (units of g)

In the performance of the test, the accelerometer is mounted with either the pendulum axis (gate position) or the output axis (pendulum position) horizontal and parallel to the axis of a precision dividing head. The positive input axis of the accelerometer is accurately positioned to 72 angles with respect to the horizontal, i.e., 0°, 5°, 10°, ..., 355°. As the head is rotated, the input axis will sense varying amounts of the earth's gravitational field. If the gate position is assumed, the applied acceleration along each axis of the accelerometer is

$$a_i = g \sin\theta \qquad a_o = g \cos\theta \qquad a_p = 0$$

in which θ is the angle between the positive input axis and the horizontal.

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The model equation will then take the form:

(2) $A = K_o + K_1\, g \sin\theta + K_2\, g^2 \sin^2\theta + K_3\, g^3 \sin^3\theta + K_4\, g^4 \sin^4\theta$
$+ K_5\, g^5 \sin^5\theta + \Delta_o\, g \cos\theta + K_{oi}\, g^2 \sin\theta \cos\theta$

Substituting the following trigonometric identities

(3) (a) $\sin^2\theta = 1/2 - 1/2 \cos 2\theta$

(b) $\sin^3\theta = 3/4 \sin\theta - 1/4 \sin 3\theta$

(c) $\sin^4\theta = 3/8 - 1/2 \cos 2\theta + 1/8 \cos 4\theta$

(d) $\sin^5\theta = 1/16 \sin 5\theta - 5/16 \sin 3\theta + 5/8 \sin\theta$

(e) $\sin\theta \cos\theta = 1/2 \sin 2\theta$

into equation (2) yields

(4) $A = (K_o + 1/2\, K_2\, g^2 + 3/8\, K_4\, g^4) + (K_1\, g + 3/4\, K_3\, g^3 + 5/8\, K_5\, g^5)$
$\sin\theta + (\Delta_o\, g) \cos\theta + (1/2\, K_{oi}\, g^2) \sin 2\theta - 1/2\, (K_2\, g^2 + K_4\, g^4)$
$\cos 2\theta - 1/4\, (K_3\, g^3 + 5/4\, K_5\, g^5) \sin 3\theta + (1/8\, K_4\, g^4) \cos 4\theta +$
$(1/16\, K_5\, g^5) \sin 5\theta$

Equation (4) may be written in Fourier series form as

(5) $A = A_o + A_1 \cos\theta + B_1 \sin\theta + A_2 \cos 2\theta + B_2 \sin 2\theta + B_3 \sin 3\theta$
$+ A_4 \cos 4\theta + B_5 \sin 5\theta$

By performing a Fourier analysis of the output waveform, the Fourier coefficients are readily obtained by a computer program. Equating the Fourier coefficients to the corresponding terms in the model equation (4), it can be seen

(6) $A_o = K_o + 1/2\, K_2\, g^2 + 3/8\, K_4\, g^4$

$B_1 = K_1\, g + 3/4\, K_3\, g^3 + 5/8\, K_5\, g^5$

$A_1 = \Delta_o\, g$

$A_2 = -1/2\, (K_2\, g^2 + K_4\, g^4)$

$B_2 = +1/2\, K_{oi}\, g^2$

$B_3 = -1/4 \, (K_3 \, g^3 + 5/4 \, K_5 \, g^5)$

$A_4 = 1/8 \, K_4 \, g^4$

$B_5 = 1/16 \, K_5 \, g^5$

The computer program is also extended to solve the equations in (6) to obtain the coefficients of the model equation.

(7) $K_o = A_o + A_2 + A_4$

$K_1 \, g = B_1 + 3B_3 + 5B_5$

$K_2 \, g^2 = -2A_2 - 8A_4$

$K_3 \, g^3 = -4B_3 - 20 \, B_5$

$K_4 \, g^4 = 8A_4$

$K_5 \, g^5 = 16B_5$

$K_{oi} \, g^2 = 2B_2$

$\Delta_o \, g = A_1$

The above equations are expressed in output units. The coefficients, except for K_1, are written in terms of g by dividing by the scale factor and appropriate power of g (equal to 1 in all cases). The resulting coefficients and their units are as follows:

K_o/K_1	units of g
K_1	output units/g
K_2/K_1	g/g^2
K_3/K_1	g/g^3
K_4/K_1	g/g^4
K_5/K_1	g/g^5
K_{oi}/K_1	g/g^2
Δ_o	radians

THE 6 POINT TEST

The six point test is employed to determine the coefficients of the accelerometer as represented by an assumed third order model equation as follows:

$$(1)\quad A = K_o + K_1 a_i + K_2 a_i^2 + K_3 a_i^3 + \Delta_o a_o + \Delta_p a_p + K_{oi} a_o a_i$$
$$+ K_{pi} a_p a_i$$

in which the symbols are the same as defined in the 72 point test (Appendix A).

The test is performed in the same manner as described in the 72 point test, but the accelerometer output readings are recorded only at six positions (0^o, 45^o, 90^o, 135^o, 180^o and 270^o).

If the gate position is assumed, the applied acceleration along each axis is

$$a_i = g \sin\theta \qquad a_o = g \cos\theta \qquad a_p = 0$$

where θ is the angle of the positive input axis with respect to the horizontal.

The model equation will then take the form

$$(2)\quad A = K_o + K_1 g \sin\theta + K_2 g^2 \sin^2\theta + K_3 g^3 \sin^3\theta + \Delta_o g \cos\theta$$
$$+ K_{oi} g^2 \sin\theta \cos\theta$$

For each of the six positions the accelerometer output will be

$$(3)\quad A_o = K_o + \Delta_o g$$

$$(4)\quad A_{45} = K_o + \frac{\sqrt{2}}{2} K_1 g + 1/2\, K_2 g^2 + \frac{\sqrt{2}}{4} K_3 g^3 + \frac{\sqrt{2}}{2} \Delta_o g + 1/2\, K_{oi} g^2$$

$$(5)\quad A_{90} = K_o + K_1 g + K_2 g^2 + K_3 g^3$$

$$(6)\quad A_{135} = K_o + \frac{\sqrt{2}}{2} K_1 g + 1/2\, K_2 g^2 + \frac{\sqrt{2}}{4} K_3 g^3 - \frac{\sqrt{2}}{2} \Delta_o g - 1/2\, K_{oi} g^2$$

$$(7)\quad A_{180} = K_o - \Delta_o g$$

$$(8)\quad A_{270} = K_o - K_1 g + K_2 g^2 - K_3 g^3$$

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From equations (3) to (8) the desired coefficients in the model equation may be solved.

(9) $K_o = 1/2\ (A_o + A_{180})$ from (3) and (7)

(10) $\Delta_o\ g = 1/2\ (A_o - A_{180})$

(11) $K_2\ g^2 = 1/2\ (A_{90} + A_{270}) - 1/2\ (A_o + A_{180})$ from (5), (8), (10)

(12) $K_3\ g^3 = (A_{90} - A_{270}) + \frac{\sqrt{2}}{2}\ (A_{90} + A_{270}) - \sqrt{2}\ (A_{45} + A_{135}) + \frac{\sqrt{2}}{2}\ (A_o + A_{180})$ from (4), (5), (6), (8), (9), and (11)

(13) $K_1\ g = -1/2\ (A_{90} - A_{270}) - \frac{\sqrt{2}}{2}\ (A_{90} + A_{270}) + \sqrt{2}\ (A_{45} + A_{135}) - \frac{\sqrt{2}}{2}\ (A_o + A_{180})$ from (5), (8), and (12)

(14) $K_{oi} = (A_{45} - A_{135}) - \frac{\sqrt{2}}{2}\ (A_o - A_{180})$ from (4), (6), and (10)

The last six equations are employed to determine the coefficients of the model equation. If the pendulum position of mounting is preferred to the gate position, the analysis remains the same but Δ_o and a_o are replaced by Δ_p and a_p.